

Decays of the B_c Meson in a Relativistic Quark–Meson Model

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Abstract

The semileptonic decay form factors of the double heavy B_c meson provide a unique opportunity to study the strong interactions between two heavy quarks. A fully relativistic model, with effective non-local quark-meson interactions, is used to compute semileptonic decay form factors, for both the B_c and a wide range of other heavy-light mesons. Using these form factors predictions for decay rates and branching ratios are obtained. The results are compared to other theoretical approaches and, where available, to experimental results. In addition the radiative decay of B_c^* is discussed.

1 Introduction

A primary goal in the study of semileptonic decays of heavy mesons is to extract the values of the CKM matrix elements. The great virtue of semileptonic decays is that the effects of the strong interaction can be separated from the effects of the weak interaction into a set of Lorentz invariant form factors [1]. Thus the theoretical problem associated with analysing semileptonic decays is essentially that of calculating the form factors.

The focus of this work is the decay of the B_c meson (for a review of the properties of this system see [2]). This system is unique among mesons made up of heavy (charm or bottom) quarks, it is the only one which is stable with respect to strong and electromagnetic interactions. Therefore, the B_c system is the only heavy meson for which form factors (albiet transition form factors rather than elastic) can be measured. These form factors then provide a unique probe of the dynamics of heavy quark systems.

There are many approaches to the calculation of decay form factors, for example, lattice QCD [3], QCD sum rules [4], and phenomenological modelling [5]. In this work a particular model with an effective quark-meson coupling is adopted. There are many models of this type [6, 7, 8, 9, 10]. The one used here has its genesis in the QCD version of the Nambu-Jona-Lasinio model [11] extended to heavy quarks [6] and is most closely related to the model used recently by Ivanov and Santorelli in a their study of pseudoscalar meson decays [12].

The advantage of this approach is that it is fully relativistic and very versatile. Quarks and mesons for all masses are treated within the same framework. For light quarks the model has the features of spontaneous chiral symmetry breaking and in the single heavy quark limit the form factor constraints of heavy quark symmetry are obtained.

Our work differs from Ivanov and Santorelli in the choice of the quark-meson vertex function and in the way that parameters are fixed. A number of heavy mesons decays not calculated in Ref.[12] are treated here. The

main new results are the extension of the model to include doubly heavy mesons, the calculation of B_c semileptonic decays and the electromagnetic vector to pseudoscalar transitions.

This paper is organized as follows: the next section introduces the model, discusses the general method of calculation, and fits the models free parameters. Sect. 3 presents the calculation of the form factors and decay rates for the semileptonic decays of a wide variety of heavy-light pseudoscalar mesons. These calculations are compared with both measured results, and other theoretical approaches. Sect. 4 presents the same set of calculations for the eight primary semileptonic decays of the B_c meson. The predictions are compared with other theoretical work, in order to highlight the differences that exist between various approaches. Sect. 5 briefly discusses the electromagnetic decays $V \rightarrow P + \gamma$ for a number of vector mesons, including the B_c^* . Sect. 6 gives conclusions and directions for future work.

2 Quark–Meson Coupling

The particular quark–meson coupling used in this work is based on an effective Lagrangian which models the interaction between mesons and quarks with a non-local interaction vertex [6, 12]. The interaction Lagrangian has the form

$$L_{\text{int}}(x) = g_M M(x) \int dx_1 dx_2 \delta\left(x - \left(\frac{1}{2} \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}\right)\right) \times f[(x_1 - x_2)^2] \bar{q}_1(x_1) \Gamma_M q_2(x_2), \quad (1)$$

where Γ_M is the Dirac matrix appropriate to the meson field M , $f[(x_1 - x_2)^2]$ is a non-local vertex function, which simulates the finite size of the meson, and q_1 and q_2 are the quark fields. A condition imposed on the vertex function is that it should render all loop diagrams UV finite. The coupling constant g_M is determined by the compositeness condition, which is the

requirement that the renormalization constant of the meson fields be zero, i.e.

$$Z_M = 1 - \frac{g_M^2}{2} \frac{d\Pi_M(p^2)}{dp^2} \bigg|_{p^2=M_M^2} = 0. \quad (2)$$

Here $\Pi_M(p^2)$ is the self energy of the meson field, given by

$$\begin{aligned} \Pi_M(p^2) = & 2N_c \int \frac{d^4k}{(2\pi)^4} f^2(Q^2) \\ & \text{tr} \left\{ \Gamma_M \frac{1}{m_1 - (\not{k} + \not{p})} \Gamma_M \frac{1}{m_2 - \not{k}} \right\}, \end{aligned} \quad (3)$$

where m_1 and m_2 are the masses of the quarks in the loop and Q is a relative momentum chosen to be $Q = k + \alpha p$ with $\alpha = \frac{m_2}{m_1+m_2}$.

The constituent quark masses in (3) are free parameters. As well, the vertex function will contain a free parameter which reflects the size of the meson. These parameters will be different for the different mesons.

The use of free constituent quark propagators in expressions like (3) can lead to a problem which reflects the lack of quark confinement in the model. If the meson mass M_M is greater than the sum of its constituent quark masses loop integrals will develop imaginary parts. This indicates a non-zero amplitude for the creation of a free quark-antiquark pair. There have been some various attempts to obviate this problem within quark-meson effective theories [7, 8, 9, 10]. Here we adopt the approach of Ref.[12] and use free propagators. The constituent quark masses are then fit to allow for the inclusion of as many mesons as possible.

In order to carry out calculations a choice must be made for the vertex function $f(q^2)$. The function that was used in this analysis was the dipole

$$f(Q^2) = \frac{\Lambda^4}{[\Lambda^2 - Q^2]^2}.$$

This choice was made for two reasons; first the form of the dipole vertex function is the same as a propagator, allowing standard Feynman parameter techniques to be used in evaluating loop integrals. Second, the vector

decay constant f_V would diverge if only a monopole vertex function was used. Since one of the primary criteria for the vertex functions is that they should render all diagrams UV finite, a function with UV fall-off as least as fast as a dipole is needed. The parameter Λ characterizes the finite size of the meson, and will be different for different mesons. To account for this the various values of Λ will be distinguished by subscripts which reflect either the meson type or the quark content, *e.g.* Λ_{B_c} and Λ_{bc} will be used interchangeably. Further, in expressions involving the vertex form factor, the same convention will be used. Note that the calculations of Ref.[12] used a Gaussian vertex function so that the parameters used there can not be compared directly with ours.

The parameters of the model were fit to the leptonic decay constants, f_P and f_V . These quantities are defined by

$$\langle 0 | -i\gamma^\mu \gamma^5 | P \rangle = if_P p^\mu, \quad (4)$$

$$\langle 0 | i\gamma^\mu | V, \epsilon \rangle = M_V^2 f_V \epsilon^\mu \quad (5)$$

where M_V is the vector meson mass. The pseudoscalar decay constant is given by the one-loop expression

$$f_P p^\mu = N_c \int \frac{d^4 k}{(2\pi)^4 i} g_P f(Q^2) \frac{\text{tr} \{ -\gamma^\mu \gamma^5 [m_1 + \not{k} + \not{p}] \gamma^5 [m_2 + \not{k}] \}}{[m_1^2 - (k+p)^2] [m_2^2 - k^2]}. \quad (6)$$

Here m_1 and m_2 refer to the masses of the quarks in the loop, this convention will be used throughout this paper. Using the dipole vertex function, combining the denominators using Feynman parameters, and performing the integration over k yields

$$f_P = g_P \frac{3\Lambda_P^4}{4\pi^2} \int D\tilde{x} \frac{x_1 [m_2(1-\sigma) + m_1\sigma]}{\Delta^2}, \quad (7)$$

with

$$\sigma = \alpha x_1 + x_2,$$

$$\begin{aligned}
\eta &= \alpha^2 x_1 + x_2, \\
\Delta &= \Lambda_P^2 x_1 + m_1^2 x_2 + m_2^2 x_3 + (\sigma^2 - \eta) M_P^2, \\
\ell &= k + \sigma p, \\
\int D\vec{x} &= \int_0^1 \left(\prod_{i=1}^3 dx_i \right) \delta \left(\sum_{i=1}^3 x_i - 1 \right),
\end{aligned}$$

where M_P is the mass of the pseudoscalar meson. Likewise the expression for the vector decay constant is

$$f_V = g_V \frac{3\Lambda_V^4}{4\pi^2 M_V^2} \int D\vec{x} x_1 \frac{m_1 m_2 + \Delta + \sigma(1 - \sigma) M_V^2}{\Delta^2}, \quad (8)$$

where the same definitions have been used, with the obvious change of M_P to M_V and Λ_P to Λ_V in the expression for Δ .

To compute the coupling constants g_P and g_V , the self energies and their derivatives must be computed. Then the compositeness condition (2) can be used to find the couplings. The self energy for a pseudoscalar meson is given by

$$\Pi_P(p^2) = \frac{3\Lambda_P^8}{2\pi^2} \int D\vec{x} x_1^3 \frac{m_1 m_2 + p^2 \sigma(1 - \sigma) + \frac{2}{3} \bar{\Delta}}{\bar{\Delta}^4}, \quad (9)$$

where $\bar{\Delta} = \Lambda_P^2 x_1 + m_1^2 x_2 + m_2^2 x_3 + (\sigma^2 - \eta) p^2$ and all the other quantities are the same as the ones defined above. The self energy for a pseudoscalar meson is given by the tensor $\Pi_V^{\mu\nu}$ which can be expressed as

$$\Pi_V^{\mu\nu}(p) = \Pi_V(p^2) g_{\mu\nu} + \bar{\Pi}_V(p^2) \frac{p_\mu p_\nu}{p^2}. \quad (10)$$

Unfortunately $\Pi_V \neq \bar{\Pi}_V$, so this does not have the proper structure for a vector propagator. This problem was solved (following [13]) by simply dropping the $\bar{\Pi}_V$ term, which would cancel out of any calculation of a physical process at one-loop order (since $\epsilon \cdot p = 0$). The relevant part of the vector meson self energy is given by

$$\Pi_V(p^2) = \frac{3\Lambda_V^8}{2\pi^2} \int D\vec{x} x_1^3 \frac{m_1 m_2 + \frac{1}{3} \bar{\Delta} + \sigma(1 - \sigma) p^2}{\bar{\Delta}^4}, \quad (11)$$

where all the quantities appearing have been defined previously.

The free parameters of the model are fit to the six values of f_P and the measured values $f_{J/\psi} = 0.1309$ and $f_\Upsilon = 0.075012$ [14]. These data, which are displayed in Table 1, fix eight free parameters. In order to reduce the number of free parameters to match the available data the value of the strange quark mass was fixed at 450 MeV and the vertex parameter for a vertex containing only u and d quarks Λ_π was taken (following [6]) to be 1 GeV. In addition the following further simplifying assumptions were made

$$\Lambda_{us} = \Lambda_{ds} = \Lambda_{ss} = \Lambda_K,$$

$$\Lambda_{uc} = \Lambda_{dc} = \Lambda_{sc} = \Lambda_D,$$

$$\Lambda_{ub} = \Lambda_{db} = \Lambda_{sb} = \Lambda_B.$$

This leaves the following parameters to be fit, m_q , m_c , m_b , Λ_K , Λ_D , Λ_B , Λ_{cc} , Λ_{bb} , and Λ_{bc} . The parameter Λ_{bc} could only be fit to a value for f_{B_c} which is not in the values listed in Table 1, hence it is retained as a free parameter, leaving eight to be fit. The fit to the remaining eight parameters is given by (all values in MeV)

$$m_{u,d} = 245,$$

$$m_c = 1800,$$

$$m_b = 5100,$$

$$\Lambda_K = 1225,$$

$$\Lambda_D = 1350,$$

$$\Lambda_B = 1500,$$

$$\Lambda_{cc} = 1420,$$

$$\Lambda_{bb} = 2900.$$

The values for the self energies, coupling constants, and leptonic decay constants arising from these parameters are displayed in Tables 2 and 3.

In order to fix Λ_{bc} a value of f_{B_c} must be given. There is no experimental value for this quantity and theoretical estimates tend to fall in the range $400 \text{ MeV} \lesssim f_{B_c} \lesssim 500 \text{ MeV}$ (see, for example, [2, 16, 17, 18, 19]). A further complication is that the mass of M_{B_c} is also not yet known very well. The current measurement [20] is $M_{B_c}^{CDF} = 6.4 \pm 0.39(stat) \pm 0.13(sys) \frac{\text{GeV}}{c^2}$, which comes from the few confirmed B_c events at the Tevatron. Theoretical results tend to lie within this range, so following the potential model prediction of [2] the mass of the B_c was chosen to be 6.25 GeV.

One general argument guides the selection of Λ_{bc} , it should lie between Λ_B and Λ_{bb} . With this in mind, and using the value for M_{B_c} above, a number of values of Λ_{bc} were tried, spanning the possible range. Fig. 1 shows the value of f_{B_c} as a function of Λ_{bc} . The value selected for use in this work was $\Lambda_{bc} = 2.3 \text{ GeV}$, which gives $f_{B_c} = 450 \text{ MeV}$, a value in the middle of the range of the theoretical predictions.

3 Semileptonic Decays of K , D , and B Mesons

The model used in this work is phenomenological but having fixed its parameters, the results for semileptonic decays are predictions. Before proceeding to decays of B_c it is important to test the model against experimental results where they are available. Therefore several semileptonic decays of K , D , and B meson are calculated. The formalism for these calculations, presented in this section, extends directly also to the calculation of B_c decay.

Some of the decays considered here have already been treated by Ivanov and Santorelli [12]. However, that work does not demonstrate the full applicability of the approach. Apart from decays to light vector mesons, the model is capable of treating virtually any semileptonic decay (with the restriction that a value for the meson mass must be supplied as input).

The amplitude A for a semileptonic decay is given by,

$$A = \frac{G_F}{\sqrt{2}} V_{QQ'} L_\mu H^\mu. \quad (12)$$

Here G_F is the Fermi constant, $V_{QQ'}$ is the relevant CKM matrix element, L_μ is the lepton current

$$L_\mu = \bar{u}_{\nu_\ell} \gamma_\mu (1 - \gamma^5) v_\ell,$$

and H^μ is the hadron current

$$H^\mu = \langle k, \epsilon | (V^\mu - A^\mu) | P \rangle, \quad (13)$$

where P is the momentum of the parent meson, k is the momentum of the daughter meson, and ϵ is the polarization, if the daughter meson is a vector. The two currents in (13) are the vector V^μ and axial A^μ . If the final state is a pseudoscalar the hadron current can be decomposed as follows,

$$\begin{aligned} \langle k | A^\mu | P \rangle &= 0, \\ \langle k | V^\mu | P \rangle &= f_+(q^2)(P + k)^\mu + f_-(q^2)(P - k)^\mu, \end{aligned}$$

where $f_+(q^2)$ and $f_-(q^2)$ are Lorentz invariant form factors. Likewise, if the final state is a vector meson,

$$\begin{aligned} \langle k, \epsilon | A^\mu | P \rangle &= f(q^2) \epsilon^{*\mu} + a_+(q^2) (\epsilon^* \cdot P) (P + k)^\mu + \\ &\quad a_-(q^2) (\epsilon^* \cdot P) (P - k)^\mu, \\ \langle k, \epsilon | V^\mu | P \rangle &= ig(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* (P + k)_\rho (P - k)_\sigma, \end{aligned}$$

where the form factors are g , f , a_+ , and a_- . In each of these expressions $q = (P - k)$ is the momentum transfer.

For a decay to a pseudoscalar meson (with mass denoted by $M_{P'}$) the differential decay rate can be reduced to [1]

$$\frac{d\Gamma}{dq} = \frac{G_F^2 |V_{QQ'}|^2 M_P^2 K^3}{24\pi^3} |f_+(q^2)|^2. \quad (14)$$

where,

$$K = \frac{M_P}{2} \sqrt{\left[1 - \frac{M_{P'}^2}{M_P^2} - y\right]^2 - 4 \frac{M_{P'}^2}{M_P^2} y}. \quad (15)$$

The lepton spectrum is given by,

$$\frac{d\Gamma}{dx} = \frac{G_F^2 |V_{QQ'}|^2 M_P^5}{16\pi^3} (1-2x) \int_0^{y_{max}(x)} ([y_{max}(x) - y] |f_+(q^2)|^2) dy, \quad (16)$$

where $y_{max}(x) = \frac{4x(x_{max}-x)}{1-2x}$ with $x_{max} = \frac{M_P^2 - M_{P'}^2}{2M_P^2}$. If the final state is a vector meson (with mass M_V) the corresponding differential decay rate is,

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{QQ'}|^2 K M_P^2 y}{96\pi^3} (|\bar{H}_+|^2 + |\bar{H}_-|^2 + |\bar{H}_0|^2), \quad (17)$$

where

$$\begin{aligned} \bar{H}_\pm &= f(q^2) \mp 2M_P K g(q^2), \\ \bar{H}_0 &= \left[\frac{M_P}{2M_V \sqrt{y}} \right] \left[\left(1 - \frac{M_V^2}{M_P^2} - y \right) f(q^2) + 4K^2 a_+(q^2) \right], \end{aligned}$$

and the final mass M_V should be substituted for $M_{P'}$ in (15). The expression for the lepton spectrum is given by

$$\begin{aligned} \frac{d\Gamma}{dx} &= \frac{G_F^2 |V_{QQ'}|^2 M_P^5}{32\pi^3} \int_0^{y_{max}(x)} \left\{ \frac{\alpha(y)}{M_P^2} y + \right. \\ &\quad \left. 2(1-2x)[y_{max}(x) - y] \beta_{++}(y) + \right. \\ &\quad \left. \gamma(y) y [2x_{max} - 4x + y] \right\}, \end{aligned} \quad (18)$$

where the following definitions were made

$$\begin{aligned} \alpha(q^2) &= |f(q^2)|^2 + \lambda |g(q^2)|^2, \\ \gamma(q^2) &= 2f(q^2)g(q^2), \\ \beta_{++}(q^2) &= \frac{1}{4M_V} \{ |f(q^2)|^2 + \lambda |a_+(q^2)|^2 - 4M_V^2 q^2 |g(q^2)|^2 \\ &\quad + 2(M_P^2 - M_V^2 - q^2) f(q^2) a_+(q^2) \}, \\ \lambda(q^2) &= (M_P^2 - M_V^2 - q^2)^2 - 4M_V^2 q^2, \\ x_{max} &= \frac{M_P^2 - M_V^2}{2M_P^2} \end{aligned}$$

Note that all of these expressions assume that lepton mass m_ℓ is zero.

Our model can be used to calculate all of these form factors. In all the following expressions the initial meson is composed of quarks with masses m_1 and m_2 and the final meson is composed of quarks with masses m_3 and m_2 (*i.e.* m_1 is the mass of the quark which decays to a new quark with mass m_3 , and m_2 is the mass of the spectator).

The form factors for decay to a pseudoscalar meson are

$$f_+(q^2) = g_P g_{P'} \frac{9\Lambda_P^4 \Lambda_{P'}^4}{\pi^2} \int D\vec{x} x_1 x_2 \frac{\chi_+ - \frac{3}{4}\bar{\Delta}(\sigma_1 + \sigma_2) + \bar{\Delta}}{\bar{\Delta}^5}, \quad (19)$$

$$f_-(q^2) = g_P g_{P'} \frac{9\Lambda_P^4 \Lambda_{P'}^4}{\pi^2} \int D\vec{x} x_1 x_2 \frac{\chi_- - \frac{3}{4}\bar{\Delta}(\sigma_1 - \sigma_2)}{\bar{\Delta}^5}. \quad (20)$$

The following definitions were made to simplify the expressions

$$\begin{aligned} \int D\vec{x} &= \int_0^1 \left(\prod_{i=1}^5 dx_i \right) \delta \left(\sum_{i=1}^5 x_i - 1 \right), \\ \Delta &= \Lambda_{P'}^2 x_1 + \Lambda_P^2 x_2 + m_3^2 x_3 + m_1^2 x_4 + m_2^2 x_5, \\ \mu_{ij} &= \frac{m_i}{m_i + m_j}, \\ \sigma_{1,(2)} &= x_{4,(3)} + \mu_{12,(23)} x_{2,(1)}, \\ \eta_{1,(2)} &= x_{4,(3)} + \mu_{12,(23)}^2 x_{2,(1)}, \\ \bar{\Delta} &= \Delta + (\sigma_1^2 - \eta_1 + \sigma_1 \sigma_2) M_P^2 \\ &\quad + (\sigma_2 - \eta_2 + \sigma_1 \sigma_2) M_{P'}^2 - \sigma_1 \sigma_2 q^2, \\ \kappa &= m_3(m_2 - m_1) + m_1 m_2 + \frac{1}{2}(M_P^2 + M_{P'}^2 - q^2), \\ \epsilon &= (\sigma_1 + \sigma_2) \sigma_1 M_P^2 + (\sigma_1 + \sigma_2) \sigma_2 M_{P'}^2 - \sigma_1 \sigma_2 q^2, \\ \zeta_1 &= m_1 m_2 - \left[\sigma_1 (\sigma_1 - 1) + \sigma_2 \left(\sigma_1 - \frac{1}{2} \right) \right] M_P^2 \\ &\quad - \sigma_2 \left(\sigma_1 + \sigma_2 - \frac{1}{2} \right) M_{P'}^2 + \sigma_2 \left(\sigma_1 - \frac{1}{2} \right) q^2, \\ \zeta_2 &= m_2 m_3 - \sigma_1 \left(\sigma_1 + \sigma_2 - \frac{1}{2} \right) M_P^2 \end{aligned}$$

$$\begin{aligned}
& - \left[\sigma_2 (\sigma_2 - 1) + \sigma_1 \left(\sigma_2 - \frac{1}{2} \right) \right] M_{P'}^2 \\
& + \sigma_1 \left(\sigma_2 - \frac{1}{2} \right) q^2, \\
\chi_{\pm} &= (\epsilon - \kappa)(\sigma_1 \pm \sigma_2) \pm \zeta_1 + \zeta_2.
\end{aligned}$$

These definitions (in addition to α and μ) will be used throughout the rest of this paper, with the obvious substitution of M_V and Λ_V for $M_{P'}$ and $\Lambda_{P'}$ when the final state is a vector meson.

The form factors for decays to vector mesons are given by

$$\begin{aligned}
g(q^2) &= g_{M_P} g_{M_V} \frac{9\Lambda_P^4 \Lambda_V^4}{\pi^2} \int D\vec{x} x_1 x_2 \\
& \frac{\sigma_2(m_2 - m_3) + \sigma_1(m_2 - m_1) - m_2}{\bar{\Delta}^5}, \tag{21}
\end{aligned}$$

$$\begin{aligned}
f(q^2) &= g_{M_P} g_{M_V} \frac{18\Lambda_P^4 \Lambda_V^4}{\pi^2} \int D\vec{x} x_1 x_2 \times \\
& \frac{1}{\bar{\Delta}^5} \left[m_1 m_2 m_3 - \frac{1}{4}(m_2 - m_1 - 2m_3)\bar{\Delta} \right. \\
& \left. + [\xi_1 + \xi_3]M_P^2 + [\xi_2 + \xi_3]M_V^2 - \xi_3 q^2 \right], \tag{22}
\end{aligned}$$

$$\begin{aligned}
a_{\pm}(q^2) &= g_{M_P} g_{M_V} \frac{18\Lambda_P^4 \Lambda_V^4}{\pi^2} \int D\vec{x} x_1 x_2 \\
& \frac{\frac{1}{2}(\beta_1 \pm \beta_2 \mp \sigma_2 m_3)}{\bar{\Delta}^5}. \tag{23}
\end{aligned}$$

The following further definitions have been made,

$$\begin{aligned}
\xi_1 &= \sigma_1(m_3 - m_2)(1 - \sigma_1) - m_1 \sigma_1^2, \\
\xi_2 &= \sigma_2(m_1 - m_2)(1 - \sigma_2) - m_3 \sigma_2^2, \\
\xi_3 &= m_2 \left[\frac{1}{2} - \frac{1}{2}(\sigma_1 + \sigma_2) + \sigma_1 \sigma_2 \right] + m_1 \sigma_1 \left(\frac{1}{2} - \sigma_2 \right) \\
& + m_3 \sigma_2 \left(\frac{1}{2} - \sigma_1 \right), \\
\beta_1 &= 2\sigma_1[m_1 \sigma_1 + m_2(1 - \sigma_1)], \\
\beta_2 &= m_2(\sigma_1 + \sigma_2 - 1 - 2\sigma_1 \sigma_2) - m_1 \sigma_1(1 - 2\sigma_2).
\end{aligned}$$

Excluding the B_c decays a total of sixteen pseudoscalar to pseudoscalar decays were considered. Due to the difficulty with confinement the corresponding number of pseudoscalar to vector decays that could be treated was only four. Table 4 shows the predictions for the decay rates and branching ratios for all of the decays considered. The values of the CKM matrix elements, and the necessary lifetimes were taken from [14].

Many of the decay rates treated in this section have been measured, hence most of the predictions can be compared to observed quantities. Table 4 shows the predicted and measured results for the branching ratios. The experimental results are taken from [14] and the errors in the predictions represent the uncertainties in the CKM matrix elements. Overall, the agreement with experiment is reasonable which increases the level of confidence in the areas where direct comparison with experiment is not possible.

Table 5 shows values of $f_+(0)$ as computed in this work and in various other theoretical approaches. The other approaches are widely varied: [21] uses the ISGW model, [5] uses the WBS model, [22] gives results from a bag model, [23] uses a Dyson-Schwinger equation approach and [3] gives lattice QCD results. Of particular interest is the work of Ivanov *et al.* [24] which uses the quark confinement model. This quark-meson model is based on similar considerations to the model used in this work so its predictions should be close to ours.

As well, the decay $B \rightarrow D + \ell^+ + \nu_\ell$ can be treated in a model independent way using the HQET. Ivanov *et al.* have shown in several papers [10, 12, 25] that quark-meson models of the type used here give the correct tree level HQET relations in the infinite mass limit. Nevertheless a direct check with finite quark mass is useful. The HQET gives the prediction [26]

$$f_+^{HQET}(q_{max}^2) = \frac{m_{B^0} + m_{D^-}}{2\sqrt{m_{B^0}m_{D^-}}} = 1.138,$$

which compares well with our value $f_+(q_{max}^2) = 1.133$.

The most important comparison that can be made is with Ref.[12]. This paper uses a different vertex function to treat B and D decays. This serves

as a check on the dependence of the model on the choice of vertex function. Apart from the case $B \rightarrow \pi + \ell + \nu$ agreement with [12] is very good. In addition [12] presents the values of $f_+(q^2)$ over the full range of q^2 . Overall agreement is good between the two calculations, Fig. 2 illustrates the agreement in the case $D^0 \rightarrow K^- + \ell^+ + \nu_\ell$. Fig. 3 shows the case $B^0 \rightarrow \pi^- + \ell^+ + \nu_\ell$, for which the agreement is better over the whole range than indicated in Table 5.

Due to lack of confinement very few pseudoscalar to vector decays can be calculated. Of the few decays treated in this work only the decay $B \rightarrow D^* + \ell + \nu_\ell$ has been studied extensively. Table 6 compares our predictions with some other calculations. Overall the agreement is reasonable.

4 Semileptonic Decays of the B_c Meson

The methods of the previous section can be directly applied to the semileptonic B_c decays. Using the procedure outline above, decay rates, lepton spectra, and branching ratios can be computed. In this work the lifetime of the B_c was taken to be 0.5 ps, which agrees with the CDF value of $\tau_{B_c}^{CDF} = 0.46_{-0.16}^{+0.18} \pm 0.03$ ps [20]. Table 7 shows $f_+(0)$, $f_+(q_{max}^2)$, the total decay rate Γ and the branching ratio for the four pseudoscalar decays. For the decays to vector mesons, values of the form factors at $q^2 = 0$ as well as total decay rates and branching ratios are displayed in Tables 8 and 9.

There are a number of other calculations of the semileptonic decays of B_c . A comparison of some results for the dominant decay modes is given in Table 10. In contrast to the situation in Sect. 3 where our quark-meson model predictions, for the most part, agreed with other models and the various other models agreed with each other, there are substantial differences between calculations of B_c decays. The clearest examples of this are the predictions for the decays to the B_s^* and J/ψ . These two decays are expected to be the most important semileptonic decay channels. However there is disagreement not only over the values of the branching ratios but also as to

which decay will be favoured. For example, the quark-meson model used in this work predicts the decay to the J/ψ to be slightly favoured over the decay to the B_s^* while the heavy quark approach used in [28] and [30] predicts the decay to B_s^* to dominate. This divergence of predictions may be expected; the heavy-heavy quark content of the B_c poses a challenge for models. Light-quark mesons may be constrained by chiral symmetry and heavy-light mesons by heavy quark symmetry. On the other hand the physics of heavy-heavy systems is less constrained by symmetries so extending models into this domain provides a severe test.

5 Electromagnetic Decays $V \rightarrow P + \gamma$

In addition to semileptonic decays the electromagnetic decays of vector mesons can be treated within our effective quark-meson coupling model. Since the amplitude involves the matrix element $\langle V|V^\mu|P \rangle$ it is clear this process will be related to the form factor $g(q^2)$. The the amplitude for this process is

$$A = -2e\epsilon^{\mu\nu\alpha\beta}\tilde{\epsilon}_\mu\epsilon_\nu p_{V\alpha}p_{P\beta}[Q_1g_1(0) + Q_2g_2(0)], \quad (24)$$

where $Q_{1,(2)}$ is the charge of $q_{1,(2)}$, and $p_{P,V}$ are the momenta of the pseudoscalar and vector mesons. The functions $g_i(0)$ are the form factors given by (21), with the appropriate masses inserted, and with $q^2 = 0$. The appropriate masses in these functions are given by the interchange of M_P and M_V and the subscript which denotes which of the quark lines the gauge field is coupled to (*i.e.* for $g_1(0)$ the appropriate expression sets $m_3=m_1$). Defining $g_{VP\gamma} = 2[Q_1g_1 + Q_2g_2]$, and summing over initial and final polarizations gives

$$|A|^2 = \frac{2\alpha\pi}{3}M_V^4 \left[1 - \frac{M_P^2}{M_V^2}\right]^2 g_{VP\gamma}^2, \quad (25)$$

where $\alpha = \frac{1}{137}$ is the fine structure constant. Standard techniques[31] yield the total rate

$$\Gamma_{VP\gamma} = \frac{\alpha}{24} M_V^3 g_{VP\gamma}^2 \left(1 - \frac{M_P^2}{M_V^2}\right)^3. \quad (26)$$

Electromagnetic decays have been the subject of several theoretical studies. As well the decay $J/\psi \rightarrow \eta_c + \gamma$ has been measured. Table 11 shows our predictions for $g_{VP\gamma}$ along with the single experimental result and the predictions of some other models. In [32] and [33] two different heavy quark approaches were used. The quark confinement model [13], which has some similarity to the quark-meson model used in this work, gives results which are quite close to ours.

There are measured branching ratios for the D^* decays, however no lifetime measurement has been made. Therefore our predictions (which do not include the lifetime) cannot be compared directly with experiment. In order to obtain branching ratios a theoretical estimate of the lifetime must be used. The quark confinement model is the ideal choice, since its predictions are closest to our work. Using the results from [13] and our predictions for the total rates (obtained from (26)) the following branching ratios are obtained:

$$\begin{aligned} BR[(D^*)^0 \rightarrow D^0 + \gamma] &= 33.0\%, \\ BR[(D^*)^+ \rightarrow D^+ + \gamma] &= 1.43\%. \end{aligned}$$

These compare well with the experimental values [14]

$$\begin{aligned} BR_{expt.}[(D^*)^0 \rightarrow D^0 + \gamma] &= 38.1\%, \\ BR_{expt.}[(D^*)^+ \rightarrow D^+ + \gamma] &= 1.1\%. \end{aligned}$$

In order to treat the electromagnetic decay $B_c^* \rightarrow B_c + \gamma$ the mass of the B_c^* meson must be specified. Theoretical estimates [2] indicate that the mass difference should be small; $M_{B_c^*} - M_{B_c} < 100$ MeV. To examine the effect of a small change in the B_c^* mass, the self energy and coupling constant

were calculated over a range of masses. These results were used to calculate $g_{B_c^* B_c \gamma}$ and are displayed in Table 12. The decay rate is shown in Fig. 4.

The radiative decay of B_c^* has not been studied extensively. A QCD sum rule approach [34], using $M_{B_c^*} = 6.6$ GeV and $M_{B_c} = 6.3$ GeV, gives the result $g_{B_c^* B_c \gamma}^{SR} = 0.270 \pm 0.095 \text{ GeV}^{-1}$. Using these masses our prediction is $g_{B_c^* B_c \gamma} = 0.2196 \text{ GeV}^{-1}$. The two values are in agreement.

6 Conclusion

A Lagrangian which models mesons in terms of an effective non-local quark-meson interaction vertex [6, 12] was extended in this paper to describe mesons such as B_c^* composed of two heavy quarks. The model has the advantage of treating all quarks (heavy and light) on the same footing, thereby permitting a unified investigation of heavy \rightarrow heavy, and heavy \rightarrow light quark decays. The model does not provide a complete dynamical description of quark interactions, meson masses can not be calculated and must be introduced as input parameters. Quark masses and the parameters associated with the quark-meson vertex were determined by fitting the pseudoscalar and vector meson decay constants f_P and f_V . Due to lack of confinement in this approach some light vector mesons had to be excluded from the analysis.

To both test the model, and demonstrate its versatility, a large number of semileptonic decays of K , D , and B mesons were analysed. Agreement with measured results and other theoretical approaches was good.

The main focus of this work was on the analysis of the semileptonic decays of the doubly-heavy B_c meson. The form factors characterizing the strong interactions of the B_c system were computed over the entire available range of momentum transfer. Using these results, decay rates and branching ratios were computed for all eight decay going to mesonic ground states.

A comparison with some other approaches highlighted the significant differences among various model predictions concerning the B_c . On very

general grounds one would expect the decays to J/ψ and $(B_s^*)^0$ states to be the most important. However, there is no agreement from models which of the channels dominates and absolute rates differ by a factor of 3 to 4. This is in contrast to the situation in K, D and B meson decays where much smaller differences between different models are found.

As a further illustration of the versatility of the model, the electromagnetic decays $V \rightarrow P + \gamma$ were investigated. A reasonable description of J/ψ and D^* radiative decay was found and the rate for $B_c^* \rightarrow B_c + \gamma$ was calculated for a range of B_c^* masses.

There is room for further analysis within the model considered in this work. Hadronic decays, such as $B_c^+ \rightarrow J/\psi + \pi^+$ can also be treated. A detailed analysis of all of these decays, combined with the results for the leptonic and semileptonic decays, could be used to make a prediction for the lifetime τ_{B_c} . A further area that needs work is the difficulty with confinement. The natural solution to this problem appears to be provided by the quark confinement model [10]. Recently Ivanov *et al.* have proposed a modification of the quark confinement model which may aid its application to heavy mesons [35].

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Table 1: Pseudoscalar Decay Constants

Meson (P)	f_P (MeV)	Source
π^+	130.7 ± 0.5	Expt. [14]
K^+	159.8 ± 1.8	Expt. [14]
D^+	$183^{+12+41+9}_{-13-0-25}$	Lattice QCD [15]
D_s^+	$229^{+10+51+3}_{-11-0-19}$	Lattice QCD [15]
B^+	$156^{+12+29+9}_{-14-0-9}$	Lattice QCD [15]
B_s^0	$177^{+11+39+13}_{-12-0-11}$	Lattice QCD [15]

Table 2: Fitted Values of Pseudoscalar Meson Properties

Meson	$\Pi(M^2)$ (GeV ²)	g_P	f_P (MeV)
π^+	0.038588	5.14916	131.06
K^+	0.068760	5.16139	160.85
D^+	0.083122	6.26671	182.80
D_s^+	0.087025	6.82039	223.81
B^+	0.134842	6.44510	142.21
B_s^0	0.136072	7.75408	187.42

Table 3: Fitted Values of Vector Meson Properties

Meson	$\Pi(M^2)$ (GeV ²)	g_V	f_V
J/ψ	0.095419	8.81505	0.131227
Υ	0.621464	7.54393	0.074796

Table 4: Predictions for Decay Rates and Branching Ratios

Parent	Daughter	Γ (ps^{-1})	$BR(\%)$	$BR_{expt.}(\%)$
K^+	π^0	3.589×10^{-6}	4.45	3.18 ± 0.08
K_S^0	π^-	7.274×10^{-6}	0.0650	0.0670 ± 0.0007
K_L^0	π^-	7.274×10^{-6}	37.6	38.78 ± 0.27
D^0	π^-	5.488×10^{-3}	0.228	0.37 ± 0.06
D^0	K^-	8.476×10^{-2}	3.52	3.50 ± 0.17
D^+	π^0	2.790×10^{-3}	0.295	0.31 ± 0.15
D^+	K^0	8.515×10^{-2}	9.000	6.8 ± 0.8
D_s^+	K^0	4.184×10^{-3}	0.195	
D_s^+	D^0	4.786×10^{-8}	2.24×10^{-6}	
B^0	π^-	6.455×10^{-5}	0.0101	0.018 ± 0.006
B^0	D^-	1.716×10^{-2}	2.68	2.00 ± 0.006
B^0	$(D^*)^-$	3.983×10^{-2}	6.21	4.60 ± 0.27
B^+	π^0	3.457×10^{-5}	5.70×10^{-3}	< 0.22
B^+	D^0	1.726×10^{-2}	2.85	1.86 ± 0.33
B^+	$(D^*)^0$	4.059×10^{-2}	6.70	5.3 ± 0.8
B_s^0	K^-	6.118×10^{-5}	9.42×10^{-3}	
B_s^0	D_s^-	1.642×10^{-2}	2.53	
B_s^0	$(D_s^*)^-$	4.185×10^{-2}	6.45	
B_s^0	B^-	2.619×10^{-8}	4.03×10^{-6}	
B_s^0	$(B^*)^-$	8.032×10^{-10}	1.24×10^{-7}	

Table 5: Comparison of this work with other approaches for the form factor $f_+(0)$. Here P is the parent meson and D is the Daughter

P	D	This work	[5]	[21]	[22]	[24]	[12]	[3]	[23]
K^+	π^0	0.9617					0.98		
D^0	K^-	0.7869	0.76	0.780	0.71	0.78	0.74	0.73	0.79
D^0	π^-	0.6292	0.69	0.681	0.8			0.65	0.86
B^0	D^-	0.7977	0.69	0.684			0.73		0.65
B^0	π^-	0.2848	0.33	0.293	0.33	0.53	0.51	0.27	0.43
B_s^0	K^-	0.2452			0.36				

Table 6: Vector Form Factors for the Decay $B \rightarrow D^* + \ell^+ + \nu_\ell$

Reference	$g(0)$ (GeV^{-1})	$a_+(0)$ (GeV^{-1})	$f(0)$ (GeV)
This Work	-0.10391	-0.09240	5.286
[5]	-0.09745	-0.09471	4.736
[27]	-0.16	-0.15	6.863
[23]	-0.09054	-0.07271	3.863

Table 7: Predictions for $B_c \rightarrow P$ Decays

P	$f_+(0)$	$f_+(q_{max}^2)$	Γ (ps^{-1})	BR (%)
B^0	0.4504	0.6816	9.7001×10^{-4}	0.049
B_s^0	0.5917	0.8075	1.8774×10^{-2}	0.94
D^0	0.1446	1.017	2.8244×10^{-5}	0.0014
η_c	0.5359	1.034	1.0355×10^{-2}	0.52

Table 8: Predictions for $B_c \rightarrow V$ Decays

V	Γ (ps^{-1})	BR(%)
$(B^*)^0$	1.048×10^{-3}	0.052
$(B_s^*)^0$	2.872×10^{-2}	1.44
$(D^*)^0$	4.739×10^{-5}	0.0024
J/ψ	2.943×10^{-2}	1.47

Table 9: Predictions for the form factors at $q^2 = 0$ for $B_c \rightarrow V$ decays.

V	$g(0)$ GeV^{-1}	$a_+(0)$ GeV^{-1}	$f(0)$ GeV
$(B^*)^0$	-0.1671	-0.0463	3.383
$(B_s^*)^0$	-0.2402	-0.0673	5.506
$(D^*)^0$	-0.0211	-0.0127	0.8296
J/ψ	-0.0784	-0.0543	4.918

Table 10: Branching Ratios for the Semileptonic Decays of the B_c .

Decay Meson	This Work	[4]	[28]	[29]	[30]
B_s^0	0.94%	$(0.68 \pm 0.23)\%$	1.35%		0.80%
$(B_s^*)^0$	1.44%	$(0.68 \pm 0.23)\%$	3.22%		2.3%
η_c	0.52%	$(0.57 \pm 0.17)\%$	0.553%	0.836%	0.15%
J/ψ	1.47%	$(0.68 \pm 0.17)\%$	2.32%	2.13%	1.5%

Table 11: The decay constant $g_{VP\gamma}$ in GeV^{-1} .

V	This Work	Expt. [14]	[23]	[32]	[33]	[13]
$(D^*)^0$	2.0321	0.5538	1.043	1.079	1.598	1.939
$(D^*)^+$	0.5224		0.1535	0.0702	0.2886	0.3950
$(D_s^*)^+$	0.2369				0.1917	0.2598
J/ψ	0.7419					
$(B^*)^0$	0.9770		0.3098	0.4177	0.5720	0.9104
$(B^*)^+$	1.7627		0.3461	0.6540	0.9701	1.618
$(B_s^*)^0$	0.6417					
Υ	0.1314					

Table 12: Calculations of B_c^* Properties and $g_{B_c^*B_c\gamma}$.

$M_{B_c^*} (GeV)$	$\Pi((M_{B_c^*})^2) (GeV^2)$	$g_{B_c^*}$	$g_{B_c^*B_c\gamma} (GeV^{-1})$
6.25	0.2843	8.627	0.3013
6.26	0.2877	8.541	0.3011
6.27	0.2912	8.455	0.3008
6.28	0.2947	8.369	0.3006
6.29	0.2984	8.283	0.3003
6.30	0.3021	8.196	0.3001
6.31	0.3059	8.109	0.2999
6.32	0.3097	8.022	0.2995
6.33	0.3137	7.934	0.2993
6.34	0.3178	7.846	0.2990
6.35	0.3220	7.758	0.2987

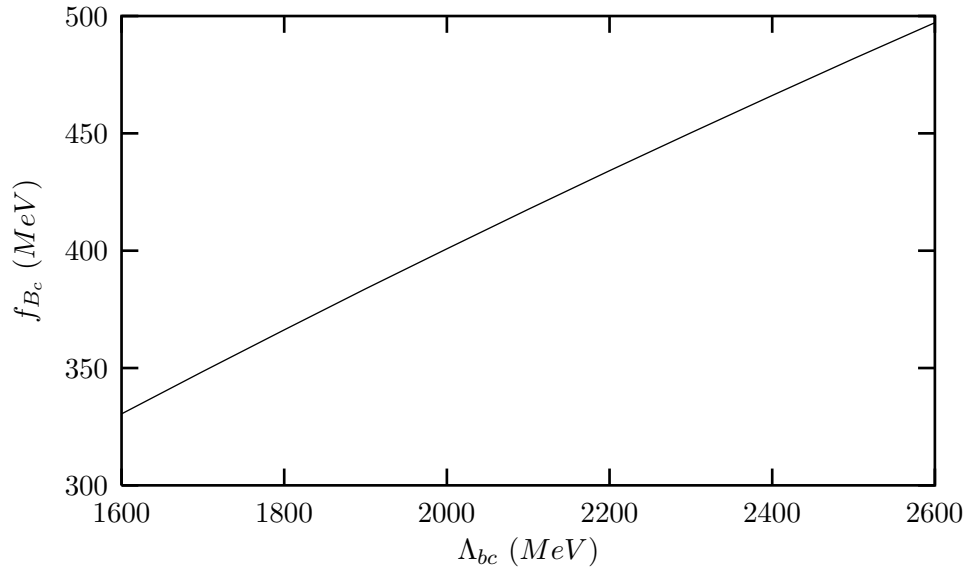


Figure 1: f_{B_c} vs. Λ_{bc}

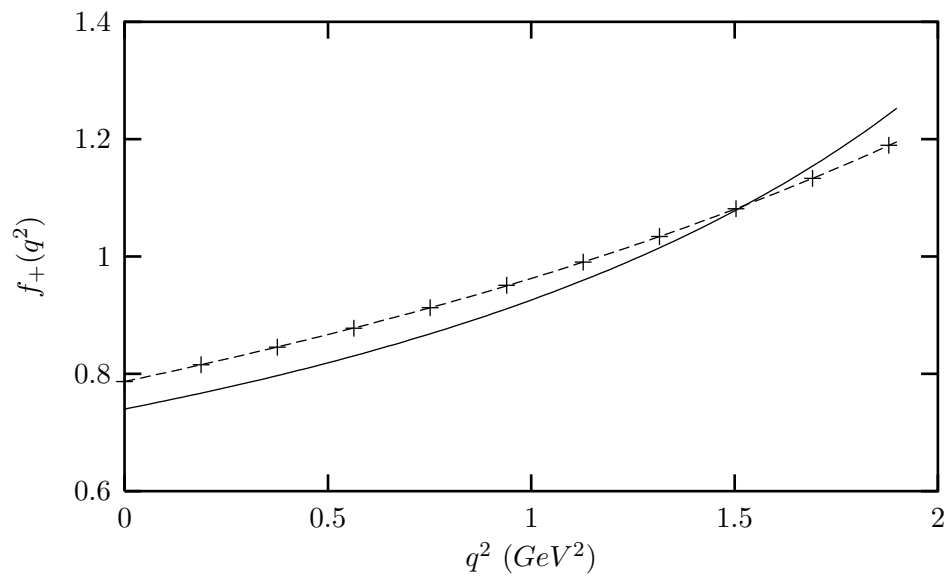


Figure 2: Form factor for the decay $D^0 \rightarrow K^-$ calculated in this work (dashed line) and in Ref. [12] (solid line).

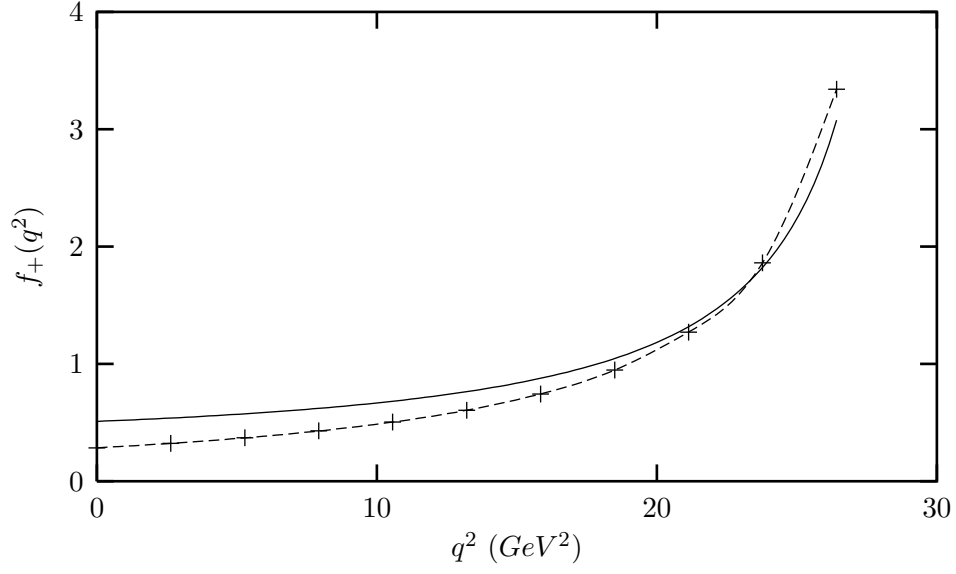


Figure 3: Form factor for the decay $B^0 \rightarrow \pi^-$ calculated in this work (dashed line) and in Ref. [12] (solid line).

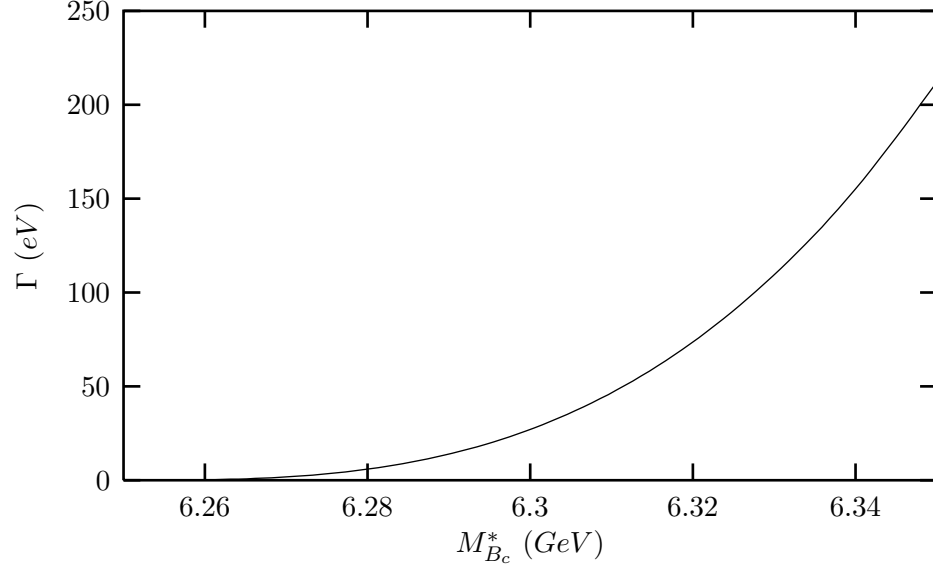


Figure 4: The decay rate $\Gamma [B_c^* \rightarrow B_c + \gamma]$ vs. $M_{B_c}^*$.